

Effect of temperature gradient on heavy quark anti-quark potential using gravity dual model

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The Quark-gluon plasma (QGP) is an expanding fireball, with finite dimensions. Given the finite dimensions, the temperature would be highest at the center, and close to the critical temperature, T_c , at the boundary, giving rise to a temperature gradient inside the QGP. A heavy quark anti-quark pair immersed in the QGP medium would see this temperature gradient. The effect of the temperature gradient on the quark anti-quark potential is analyzed using a gravity dual model. The resulting modification to the potential due to the temperature gradient is seen to have a L^{-2} correction term. This could be a possible fallout of the breaking of conformal invariance at finite temperature.

Keywords : QGP, AdS-CFT, gravity dual model, potential, Wilson loop.

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I. INTRODUCTION

The suppression of J/ψ and Υ are two prominent signatures to detect the presence of QGP, as well as to study its properties [1–4]. In [5, 6], it was shown that at high temperatures, the heavy quark anti-quark separation increases, which leads to an increase in formation time for both J/ψ and Υ . This in turn results in a drastic reduction in suppression due to the color screening mechanism [5, 6]. The temperature inside the QGP medium would be non-uniform spatially. As the heavy quark anti-quark separation increases, the heavy quark and anti-quark in the QGP medium will be subjected to two different temperatures. In this letter, we explore the effect of this temperature gradient in the QGP medium on the potential between the heavy quark and anti-quark using a gravity dual model. Accurate determination of heavy quark anti-quark potential is essential to determine the J/ψ and Υ suppression precisely.

The gravity dual model in the form of AdS-CFT correspondence, was first proposed by Maldacena in his seminal work [7, 8]. It was shown that the strong coupling in the framework of Quantum Chromo Dynamics (QCD), corresponds to a weak coupling in the gravity dual domain, and hence calculations can

be more easily performed in the gravity dual domain. With the increase in the separation between the quark and anti-quark, the coupling constant would become larger, and hence, the potential cannot be calculated accurately using perturbative QCD. The gravity dual model provides the ideal mathematical framework to determine the effect of a temperature gradient on the heavy quark anti-quark potential, when the separation between them is large. However, the gravity model in AdS_5 space is dual to $\mathcal{N} = 4$ supersymmetric Yang Mills Lagrangian, and hence our calculation would be for a $\mathcal{N} = 4$ supersymmetric Yang Mills Lagrangian instead of the QCD Yang Mills Lagrangian.

The organization of the rest of the letter is as follows. Section II frames the problem in both the Wilson loop domain and in the gravity dual domain. The case of constant finite temperature is also treated in Sec. II. The effect of temperature gradient on heavy quark anti-quark potential is formulated and solved in Sec. III. The conclusion is finally drawn in Sec. IV.

II. WILSON LOOP AND FORMULATION

The Wilson loop that has been used to model the scenario of the temperature gradient is depicted in Fig. 1. The relation of the Wilson loop to the heavy quark anti-quark potential could be modeled as [9],

$$\langle W(C) \rangle = e^{-V_{Q\bar{Q}}\beta}, \quad (1)$$

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where $\beta = \frac{1}{\Theta}$, and Θ is the temperature of the system. $V_{Q\bar{Q}}$ is the potential between the heavy quark and anti-quark. From Maldacena's conjecture, the vacuum expectation value of the Wilson loop for a $\mathcal{N} = 4$ supersymmetric Yang Mills Lagrangian, could be equated to the string action $= S = \frac{1}{2\pi\alpha'} \int_{\Sigma} d\sigma d\tau \sqrt{G_{MN} \partial_{\alpha} X^M \partial_{\beta} X^N}$, where Σ is the domain covered by the Wilson loop, which lives at the Minkowski boundary of the AdS_5 space. In the integral, $\tau = i\beta$, and σ takes the value from 0 to L .

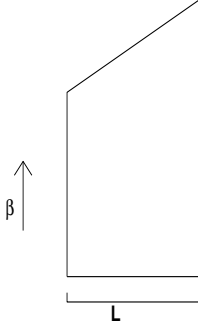


FIG. 1: Wilson loop to model the potential of a heavy quark anti-quark pair immersed in a QGP medium with a temperature gradient.

For the Wilson loop shown in Fig. 1, one can break the corresponding string action into 2 parts:

$$S = \frac{1}{2\pi\alpha'} \int_{\Sigma_1} d\sigma d\tau \sqrt{G_{MN} \partial_{\alpha} X^M \partial_{\beta} X^N} \quad (2)$$

$$+ \frac{1}{2\pi\alpha'} \int_{\Sigma_2} d\sigma d\tau \sqrt{G_{MN} \partial_{\alpha} X^M \partial_{\beta} X^N} \quad (3)$$

$$= S_1 + S_2. \quad (4)$$

The two string actions S_1 and S_2 correspond to the two Wilson loops L_1 and L_2 (Fig. 2), which make up the original Wilson loop L . Varying S , we get,

$$\delta S = \delta S_1 + \delta S_2 = 0.$$

A sufficient, albeit stricter condition for $\delta S = 0$ is that the individual variation in action be zero, i.e. $\delta S_1 = \delta S_2 = 0$. This allows the two loops L_1 and L_2 to be treated independently.

We give the intuition behind treating the two actions S_1 and S_2 separately, by analyzing the equivalent Wilson loop domain. Let us consider the

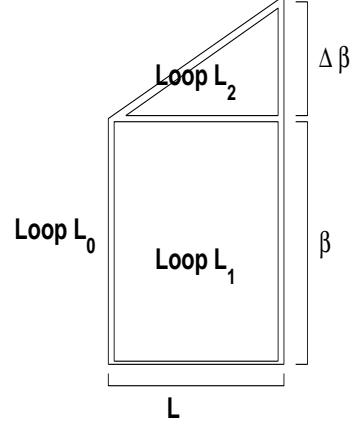


FIG. 2: The two Wilson loops L_1 and L_2 , with the original loop L_0

zero temperature case, where the "temporal" axis is time. As the time interval "T" becomes large, only the smallest eigenvalue (the ground state) survives in the Wilson loop. To see why, consider the Wilson loop, W , for a rectangular loop $L \times T$. We treat the Wilson loop in axial gauge, with A along the time axis $= 0$. This gives [10],

$$W = \langle \text{tr} \Psi(0) \Psi^{\dagger}(T) \rangle,$$

where,

$$\Psi(t) = \exp(-i \int_0^L A_{\mu}(z, t) dz).$$

Inserting a complete set of intermediate eigenstates $|n\rangle$,

$$\begin{aligned} W &= \sum_n \langle \Psi(0) | n \rangle \langle n | \Psi^{\dagger}(T) \rangle \\ &= \sum_n | \langle \Psi(0) | n \rangle |^2 e^{-E_n T}, \end{aligned}$$

where E_n is the energy of the state $|n\rangle$. As $T \rightarrow \infty$, only E_0 survives.

Decompose the rectangular region $L \times T$ into $L \times T_1$ and $L \times T_2$, with $T_1 + T_2 = T$. The domains, Ω_1 and Ω_2 denote the two rectangular regions $L \times T_1$ and $L \times T_2$. Under the conditions when only one eigenvalue survives or is the dominant contributor,

$tr \left(\exp \left\{ (-i \int_{\Omega_1} A_\mu dx^\mu) + (-i \int_{\Omega_2} A_\mu dx^\mu) \right\} \right)$
 $\approx tr \left(\exp(-i \int_{\Omega_1} A_\mu dx^\mu) \right) tr \left(\exp(-i \int_{\Omega_2} A_\mu dx^\mu) \right)$,
 where $A_\mu = A_\mu^{at^a}$. This implies that the two actions corresponding to the two Wilson loops can be treated separately as an approximation. In Maldacena's derivation [7], it can be explicitly be seen that if the action is broken into two actions corresponding to two loops $L \times T_1$ and $L \times T_2$, the final result is the same as the single loop $L \times T$. Extending to the finite temperature case, if only the ground state contributes towards the action S , i.e. when β is large, then S_1 and S_2 may be treated independently as an approximation.

The $AdS_5 \times S_5$ metric at finite temperature is given by [8, 9]

$$ds^2 = \alpha' \left[\frac{U^2}{R^2} (-H dt^2 + dx_{||}^2) + \frac{R^2}{U^2} \left(\frac{1}{H} dU^2 + U^2 d\Omega_5^2 \right) \right].$$

Here,

$H = 1 - \frac{K^4}{U^4}$, with $K^4 = \frac{2^7}{3} \pi^4 g^2 \mu$. The free energy density, μ , is related to temperature Θ and is given by $\mu = \frac{4\pi^2}{45} \Theta^4$.

$R^4 = 4\pi g N$, with N large.

Using standard techniques [7, 8], and after subtracting the contribution of the self energy of the heavy quark and anti-quark system to the action, it can be seen that

$$S_1 = 2\beta \frac{I(R, y, \omega)}{2\pi L} \left(\int_1^\infty dy \left(\frac{y^2}{\sqrt{y^4 - 1}} - 1 \right) - 1 \right) \\ = \beta \frac{I(R, y, \omega)}{2\pi L} \left(\frac{-(2\pi)^{3/2}}{\Gamma(1/4)^2} \right), \quad (5)$$

where (the integral $I(R, y, \omega)$ has been evaluated in [9]),

$$I(R, y, \omega) = 2R^2 \int_\omega^\infty \frac{dy}{\sqrt{y^4 - \omega^4} \sqrt{y^4 - 1}} \\ = \frac{2R^2}{4\sqrt{(2\omega^3)}} \left[K \left(\sqrt{\frac{\omega/2+1/(2\omega)+1}{(\omega+1/\omega)}} \right) - K \left(\sqrt{\frac{\omega/2+1/(2\omega)-1}{\omega+1/\omega}} \right) \right],$$

with K being the complete elliptic integral of the first kind;

$y = U/U_0$, with U_0 being the minimum of U ; and

$\omega = K/U_0$.

III. THE TEMPERATURE GRADIENT CASE

For the purpose of calculation of the potential in the presence of a temperature gradient, we use the same metric, namely,

$$ds^2 = \alpha' \left[\frac{U^2}{R^2} (-H dt^2 + dx_{||}^2) + \frac{R^2}{U^2} \left(\frac{1}{H} dU^2 + U^2 d\Omega_5^2 \right) \right].$$

The values of $h_{\alpha\beta} = G_{MN} \partial_\alpha X^M \partial_\beta X^N$ with $t = i\beta$, are:

$$h_{00} = \frac{U^2}{R^2} H; h_{11} = \frac{U^2}{R^2} + \frac{R^2}{U^2 H} U'^2; \\ \text{and } h_{01} = h_{10} = 0;$$

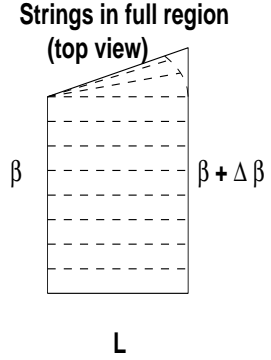


FIG. 3: Dashed lines represent the string contours (top view) in the entire region covered by the full Wilson loop, as the string traverses the worldsheet area.

The strings in the full region covered by the Wilson loop and the strings in the triangular region are shown in Figs. 3 and 4 respectively. The string action for the triangular region, with $\tau = \beta$ and $\sigma = x$, would be given by,

$$S_2 = \frac{1}{2\pi} \int_{\Sigma_2} d\sigma d\tau \sqrt{U'^2 + \frac{U^4}{R^4} H},$$

where Σ_2 represents the triangular region shown in Fig. 4. As can be seen in Fig. 4, the string contours

Triangle Approximated as a Sector

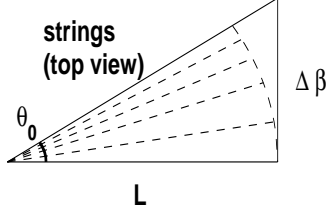


FIG. 4: Dashed lines represent the string contours in top view in the triangular region. The triangular region is approximated as a sector of a circle

in triangular region are most naturally represented in polar coordinates. Converting to polar coordinates,

$$S_2 = \frac{1}{2\pi} \int d\theta dr r \sqrt{U'^2 + \frac{U^4}{R^4}} H, \quad (6)$$

where, U and H are now expressed in polar coordinates. From the symmetry of the problem, U would depend on r alone. The Lagrangian explicitly depends on r , and hence the Hamiltonian would also explicitly depend on r . The Hamiltonian, $H^a =$

$$\frac{rU^4/R^4 H}{\sqrt{U'^2 + U^4/R^4 H}}.$$

We express the above Hamiltonian as $H^a = rH_0^a$. We have separated out the constant H_0^a from the r dependent Hamiltonian. If U_0 is the value of U at the minima, then $H_0^a = \frac{U_0^2 \sqrt{H}}{R^2}$. This gives

$$rH_0^a = \frac{rU^4/R^4 H}{\sqrt{U'^2 + U^4/R^4 H}}.$$

Substituting H_0^a and solving for U ,

$$\frac{U'}{U_0} = \frac{U_0}{R^2} \sqrt{\frac{U^4}{U_0^4} - \frac{K^4}{U_0^4}} \sqrt{\frac{U^4}{U_0^4} - 1}.$$

It is seen that the explicit r dependence is eliminated. Again, defining $y = U/U_0$, and $\omega = K/U_0$,

$$y' = \frac{U_0}{R^2} \sqrt{y^4 - \omega^4} \sqrt{y^4 - 1}.$$

The variable ω is actually a function of r , as temperature is a function of r . However, if the temperature gradient is small, i.e. the temperature variation between the two quarks is small, this can be ignored, and ω can be taken independent of r . This gives

$$\int_{L/2}^L dr = L/2 = \frac{R^2}{U_0} \int_{\omega}^{\infty} \frac{dy}{\sqrt{y^4 - \omega^4} \sqrt{y^4 - 1}}, \quad (7)$$

or

$$U_0 = \frac{2R^2}{L} \int_{\omega}^{\infty} \frac{dy}{\sqrt{y^4 - \omega^4} \sqrt{y^4 - 1}} = I(R, y, \omega)/L, \quad (8)$$

with $I(R, y, \omega)$ as defined earlier.

Substituting the expression for y' and dr in the action in Eq. 6, and noting that the integrand is independent of θ , we get the result,

$$S_2 = \frac{U_0}{2\pi} \theta_0 \left(\int_1^{U/U_0} dy r(y) \frac{y^2}{\sqrt{y^4 - 1}} + \int_{U/U_0}^1 dy r(y) \frac{y^2}{\sqrt{y^4 - 1}} \right).$$

The string goes from ∞ to U_0 and then from U_0 to ∞ . Hence the integration is divided into two limits, 1 to U/U_0 and U/U_0 to 1. The heavy quark anti-quark system self energy needs to be subtracted. The self energy contribution to action (and remembering that U is now in polar coordinates) $= S_{SE} \equiv \frac{1}{2\pi} \int_0^{\theta_0} r d\theta \int_{U_0}^U dU + \frac{1}{2\pi} \int_0^{\theta_0} r d\theta \int_U^{U_0} dU + L\theta_0 \frac{U_0}{2\pi} = U_0\theta_0 \int_1^{U/U_0} r dy + U_0\theta_0 \int_{U/U_0}^1 r dy + U_0L\theta_0 \frac{U_0}{2\pi}$. After subtracting S_{SE} from S_2 , S_2 becomes,

$$S_2 = \frac{U_0}{2\pi} \theta_0 \left(\int_1^{U/U_0} dy r(y) \left(\frac{y^2}{\sqrt{y^4 - 1}} - 1 \right) + \int_{U/U_0}^1 dy r(y) \left(\frac{y^2}{\sqrt{y^4 - 1}} - 1 \right) - L \frac{U_0}{2\pi} \right).$$

As y ranges from U/U_0 to 1, r ranges from 0 to $L/2$, and as y ranges from 1 to U/U_0 , r ranges from $L/2$ to L . Substituting the expression for U_0 and

recognizing that for small θ_0 , $\theta_0 = \frac{\Delta\beta}{L}$,

$$S_2 = \frac{I(R, y, \omega)}{2\pi L} \frac{\Delta\beta}{L} \left\{ \int_1^{U/U_0} dy r(y) \left(\frac{y^2}{\sqrt{y^4 - 1}} - 1 \right) + \int_{U/U_0}^1 dy r(y) \left(\frac{y^2}{\sqrt{y^4 - 1}} - 1 \right) - L \frac{U_0}{2\pi} \right\}.$$

Integrating by parts,

$$S_2 = \frac{I(R, y, \omega)\Delta\beta}{2\pi L^2} \times \left\{ \left[r \int dy \left(\frac{y^2}{\sqrt{y^4 - 1}} - 1 \right) \right]_{y=1}^{y=\infty} + \left[r \int dy \left(\frac{y^2}{\sqrt{y^4 - 1}} - 1 \right) \right]_{y=\infty}^{y=1} - L \right\} - \frac{I(R, y, \omega)\Delta\beta}{2\pi L^2} 2 \int_1^\infty dy \int dy \left(\frac{y^2}{\sqrt{y^4 - 1}} - 1 \right).$$

Solving the integrals,

$$S_2 = I(R, y, \omega)\Delta\beta \left\{ \frac{1}{2\pi L} \left(\frac{-2\pi^{3/2}}{\Gamma(1/4)^2} \right) - \frac{C}{2\pi L^2} \right\}, \quad (9)$$

with C being a positive constant and is equal to $2 \int_1^\infty dy \int dy \left(\frac{y^2}{\sqrt{y^4 - 1}} - 1 \right)$.

The L^{-2} correction is seen. A pure L^{-1} term indicates conformal invariance. The L^{-2} term indicates the conformal invariance is broken. The result is not surprising as conformal invariance is broken at finite temperatures due to non-zero mass effects.

From Eqs. 5 and 9, the final potential between the heavy quark and anti-quark is,

$$\begin{aligned} < V_{Q\bar{Q}}(L, T) > = S/\beta \\ &= I(R, y, \omega) \left[\frac{1}{2\pi L} \left\{ \left(2 + \frac{\Delta\beta}{\beta} \right) \left(\frac{-(2\pi)^{3/2}}{\Gamma(1/4)^2} \right) \right\} - \frac{\Delta\beta/\beta}{2\pi L^2} C \right]. \quad (10) \end{aligned}$$

The zero temperature gradient case is obtained by simply assigning $\Delta\beta = 0$. This gives,

$$< V_{Q\bar{Q}}(L, T) > = I(R, y, \omega) \left[\frac{1}{2\pi L} \left\{ \frac{-(2\pi)^{3/2}}{\Gamma(1/4)^2} \right\} \right],$$

which is conformally invariant.

IV. CONCLUSIONS

The effect of a temperature gradient on the heavy quark anti-quark potential has been analyzed and calculated. The modification in potential due to the temperature gradient is seen to be proportional to $\Delta\beta$. A term proportional to L^{-2} is also seen. This term is not conformally invariant. A non conformally invariant term is a possibility at finite temperatures, since conformal invariance is broken at finite temperatures. This calculation, however, provides the heavy quark anti-quark potential for a $\mathcal{N} = 4$ supersymmetric Yang Mills Lagrangian, instead of the QCD Lagrangian.

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